International Journal of Theoretical Physics, Vol. 31, No. 1, 1992

Spontaneous Symmetry Breaking in Static Robertson–Walker Space-Time with Background Charge

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Received May 11, 1991

The finite-temperature $\lambda\varphi^4$ theory of static Robertson–Walker (RW) space-time is extended to a case with background charge. In contrast to earlier work on static RW space-time, the curvature term is retained and its effect on the effective potential and phase transition are explicitly calculated. The spontaneous symmetry breaking aspects and its dependence on various factors are discussed.

1. INTRODUCTION

Recently the study of finite-temperature field theory has become increasingly important because of its possible application in various fields of physics. In particular, $\lambda\varphi^4$ theory has been investigated by several authors (Connor *et al.*, 1983; Kennedy, 1981; Denardo and Spallucci, 1981; Semenoff and Weiss, 1985*a,b*; Anderson and Holman, 1986; Roy *et al.*, 1989). Field theory in an RW background metric is of special interest with regard to cosmological phase transitions.

In this paper we choose a static RW space-time with a net interacting background charge and study finite-temperature effects on $\lambda\varphi^4$ theory by adopting the GEP (Gaussian effective) approach.

The justification for using the GEP is that, being nonperturbative, it has several advantages over the loop-expansion method (Stevenson, 1984*a,b*, 1985) and contains one-loop as well as $1/N$-expansion results in limiting cases for scalar fields (Stevenson, 1984*a,b*, 1985; Hajj and Stevenson, 1988).

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0020-7748/92/0100-0137$06.50/0

Of the two versions of renormalization in GEP, we choose the "autonomous" version. The other version becomes "precarious" in 3 + 1 dimensions and faces a triviality problem. It also produces strange results at high temperature (Stevenson, 1984*a*,*b*, 1985; Hajj and Stevenson, 1988; Tarrach, 1986).

In this report we have excluded the open-universe case in RW spacetime since it has already been reported (Roy *et al.*, 1989; Tarrach, 1986) that finite-temperature GEP results are not much different from those for flat space.

Also in the present case we restrict ourselves to the case of a single chemical potential. However, the inclusion of more than one μ is straightforward. We present the derivation of the finite-temperature GEP in Section 2.

Section 3 deals with the behavior of the GEP at $T \neq 0$, its phase transitional aspects, and its dependence on various parameters. Section 4 includes remarks and discussions.

2. CALCULATION OF FINITE-TEMPERATURE GEP

We start with the line element (Hawking and Ellis, 1973)

$$ds^2 = dt^2 - a^2[d\chi^2 + f^2(\chi)(d\theta^2 + \sin^2\theta\, d\varphi^2)] \tag{2.1}$$

where $f(\chi)$ is determined by the constant space curvature k, which is normalized to 1 in our case (static universe), and

$$f(\chi) = \sin(\chi), \qquad 0 \leq \chi \leq \pi \quad \text{for } k = 1 \tag{2.2}$$

The dimension of the spatial line element is carried by a^2. The introduction of the conformal time $\eta = a^{-1}t$ and use of coordinates η, χ, θ, φ enables one to write the following nonzero metric tensor components (Birell and Davies, 1982):

$$\begin{gathered} g^{00} = g^{11} = a^{-2}, \qquad g^{22} = -[af(\chi)]^{-2} \\ g^{33} = -[af(\chi)\sin\theta]^{-2} \end{gathered} \tag{2.3}$$

Let us consider the following N scalar fields in curved space:

$$\mathscr{L} = -\tfrac{1}{2}[-g]^{1/2}[g^{\gamma\nu}(\nabla_\gamma{}^{\varphi}{}_{a})(\nabla_\nu{}^{\varphi}{}_{a}) - (m^2 + \xi R)\varphi^2 - 2\lambda(\varphi^2)^2] \tag{2.4}$$

where $a = 1, 2, 3, \ldots, N$; ξ is a numerical factor; $\xi R\varphi^2$ denotes the coupling between gravitational and scalar fields; $g = \det g_{\mu\nu} = -[a^4 f^2(\chi)\sin\theta]^2$; and ∇_ν is the covariant derivative.

The free field is given by (Birell and Davies, 1982)

$$\varphi_a = \int d\bar{\mu}(k)\, [a_k u_k(\chi) + a_k^+ u_k^*(\chi)] \tag{2.5}$$

a_k^+ , a_k are the creation and annihilation operators. The function $u_k(\chi)$ can be expressed as

$$u_k(\chi) = y_k(x)\chi_k(\eta)a^{-1} \tag{2.6}$$

with

$$\chi_k(\eta) = 2(w_k)^{-1/2} e^{-iw_k\eta} \tag{2.7}$$

and $y_k(x)$ is a solution of

$$\Delta^3 y_k(x) = -(k^2 - 1)y_k(x) \tag{2.8}$$

and has the form

$$y_k(x) = \pi_{kl}^+ Y_{lm}(\theta, \varphi) \tag{2.9}$$

where $Y_{lm}(\theta, \varphi)$ denotes spherical harmonics and the properties of π_{kl}^+ can be found in Parker and Fulling (1974) and Bander and Itzykson (1966). For static RW space-time

$$K = (k, l, m), \qquad k = 1, 2, \ldots, \quad l = 0, 1, \ldots, k - l \tag{2.10}$$

and

$$m = -l, -l+1, \ldots, l$$

The measure for a static universe ($k = 1$) is given by

$$d\bar{\mu} = \sum_{klm}$$

Now we take the trial fields in the form

$$\varphi_i(x) = (\varphi_0)_i + (\varphi_\Omega(x))_i \tag{2.11}$$

where $(\varphi_0)_i$ is the constant background field and $(\varphi_\Omega(x))_i$ is the ith free quantum field with trial mass Ω. The ground state corresponding to $\varphi_\Omega(x)$ is $|0\rangle_\Omega$, which satisfies $a_k|0\rangle_\Omega = 0$. Now equation (2.8) gives

$$w_k^2 = a^2\Omega^2 - 1 + k^2 \tag{2.12}$$

The Hamiltonian corresponding to (2.4) is given by

$$H=\tfrac{1}{2}a^2f^2(\chi)\sin\theta\Bigg[(\partial_n\varphi_\alpha)(\partial_n\varphi_\alpha)+h^{ij}(\partial_i\varphi_\alpha)(\partial_j\varphi_\alpha)$$
$$+a^2\left(m^2+\frac{1}{a^2}\right)\varphi^2+2a^2\lambda(\varphi^2)^2\Bigg] \tag{2.13}$$

We introduce a parameter μ by replacing ∂_n by $\partial_n+i\mu$ [the method followed by Kapusta (1981) in flat space]. Here we consider a single μ, i.e., we take $\alpha=1,2$ (Haber and Weldon, 1982) in equation (2.13). However, the extension to more than one μ is straightforward. In general the $(\varphi_0)_i$ are not necessarily the same, but for the sake of simplicity we choose $(\varphi_0)_2=0$ and hence we shall write $(\varphi_0)_i=\varphi_0$. Also we choose the same mass parameter Ω for the $\varphi_i(x)$. Calculating $\langle 0|H|0\rangle_\Omega$ and dropping the unimportant factor $f^2(\chi)\sin\theta$ (Roy *et al.*, 1989; Tarrach, 1986), one gets the GEP V_G and the result is

$$V_G(\varphi_0,\Omega,\mu)=\Bigg[I_1^{\mathrm{FT}}+\tfrac{1}{2}(a^2m^2-M^2)+\frac{a^4}{2}m^2\varphi_0^2+\lambda a^4\varphi_0^4$$
$$-\frac{\mu^2}{2}\varphi_0^2+3\lambda(I_0^{\mathrm{FT}})^2+6\lambda a^2\varphi_0^2 I_0^{\mathrm{FT}}\Bigg] \tag{2.14}$$

For evaluating $\langle 0|H|0\rangle_\Omega$ we have used the following renormalization conditions (Tarrach, 1986)

$$\int d^3x\, h^{1/2}y_k(x)y_{k'}^*(x)=\delta(k,k')$$
$$[a_k,a_{k^+}]=\delta(k,k') \tag{2.15}$$
$$\int \overline{d\mu}(k')\,\delta(k,k')f(k')=f(k)$$

In equation (2.14) we have written $M^2=a^2\Omega^2-1$ and have redefined the bare parameter μ so as to absorb the $1/a^2$ term in it. The general form of the integrals appearing in equation (2.14) is given by

$$I_N^{\mathrm{FT}}=\int (dk)_\Omega\,[w_k^2]^N \tag{2.16}$$

where w_k is given by equation (2.12).

Following the method of introduction of μ by Kapusta (1981) in flat space and using equation (2.11) and taking the appropriate measure in RW space-time, we obtain the covariant form (Raditi, 1986) of I_1^{FT},

$$I_1^{\mathrm{FT}}=\int_{-\infty}^{\infty}\frac{dk_0}{2\pi}\int d\bar{\mu}\ \{\ln[(w_n-i\mu)^2+w_k^2] + \ln[(w_n+i\mu)^2+w_k^2]\}\,|y_k(x)|^2 \tag{2.17}$$

The integral over the momentum component becomes (Anderson and Holman, 1986) $1/\beta\sum_{n=-\infty}^{\infty}$ and w_n is replaced by $2\pi n/\beta$ and the measure $\int d\bar{\mu}\ |y_k(x)|^2$ is given by $(1/4\pi^2)\sum_{k=1}^{\infty}k^2$ (Tarrach, 1986). So equation (2.17) becomes

$$I_1^{\mathrm{FT}}=\frac{1}{\beta}\sum_{k=1}^{\infty}\frac{k^2}{4\pi^2}\sum_{n=-\infty}^{\infty}\{\ln[(w_n-i\mu)^2+k^2+M^2] + \ln[(w_n+i\mu)^2+k^2+M^2]\} \tag{2.18}$$

with $M^2=a^2\Omega^2-1$. We write $I_1^{\mathrm{FT}}=I+I'$, with

$$I=\frac{1}{\beta}\sum_{k=1}^{\infty}\frac{k^2}{4\pi^2}\sum_{n=-\infty}^{\infty}\ln[(w_n-i\mu)^2+k^2+M^2] \tag{2.19a}$$

and

$$I'=\frac{1}{\beta}\sum_{k=1}^{\infty}\frac{k^2}{4\pi^2}\sum_{n=-\infty}^{\infty}\ln[(w_n+i\mu)^2+k^2+M^2] \tag{2.19b}$$

Using the representation (Randjibar Daemi *et al.*, 1984)

$$\ln X=-\frac{d}{dS}X^{-S}\Big|_{S=0} \tag{2.20}$$

and then using the Γ-function representation of X^{-S}, we get

$$I=-\frac{1}{4\pi^2\beta}\frac{d}{dS}\frac{1}{\Gamma(S)}\sum_{k=1}^{\infty}k^2 \times\sum_{n=-\infty}^{\infty}\int_0^{\infty}\exp\{-[k^2+M^2+(w_n-i\mu)^2]t\}t^{S-1}\,dt\,|_{S=0} \tag{2.21}$$

Now expressing (2.21) in terms of the theta function θ_3 and σ_3, we have

$$I=-\frac{1}{4\pi^2\beta}\frac{d}{dS}\frac{1}{\Gamma(S)}\int_0^{\infty}t^{S-1}\,e^{-a^2\Omega^2t}\sigma_3(t)(-i\tau)^{1/2}\theta_3(z,\tau)\,|_{S=0} \tag{2.22}$$

where $\sigma_3(t)$ is given by (Randjibar Daemi *et al.*, 1984)

$$\sigma_3(t) = \sum_{k=1}^{\infty} k^2 \exp[-(k^2-1)t]$$

and

$$Z = -i\mu\beta/2\pi \qquad \text{and} \qquad i/\tau = 4\pi t/\beta^2$$

Obtaining a similar expression for I' and using the properties of the θ_3 function (Gradshteyn and Ryzik, 1982), we have

$$I_1^{\mathrm{FT}} = I_1^{\beta} + I_1(\Omega) \tag{2.23}$$

with

$$I_1(\Omega) = \frac{1}{2\pi^2} \sum_{k=1}^{\infty} k^2 (k^2 + M^2)^{1/2} \tag{2.24}$$

and

$$I_1^{\beta} = -\frac{1}{2\pi^{5/2}} \frac{d}{dS} \frac{1}{\Gamma(S)} \int_0^{\infty} \sigma_3(t) t^{S-3/2} e^{-a^2\Omega^2 t} \times \sum_{n=1}^{\infty} e^{-n^2\beta^2/4t} \cosh \mu\beta n \, dt \,|_{S=0} \tag{2.25}$$

It is worth mentioning that even in our case I_1^{FT} splits into temperature-independent, $I_1(\Omega)$, and temperature-dependent, I_1^{β}, parts, as in the case without μ in curved space (Roy *et al.*, 1989) and flat space (Hajj and Stevenson, 1988). Using the properties of $\sigma_3(t)$ and evaluating standard integrals (Gradshteyn and Ryzik, 1982) appearing in (2.25), we obtain

$$I_1^{\beta} = -\frac{1}{\pi^2} \sum_{n=1}^{\infty} \left[\frac{a^2\Omega^2}{n^2\beta^2} K_2(a\Omega\beta n) + \frac{a\Omega}{2n\beta} K_1(a\Omega\beta n) + \frac{K_0(a\Omega\beta n)}{8} \right] \cosh \mu\beta n \tag{2.26}$$

Now using the integral representation of $K_\nu(z)$, we get, after a few steps of straightforward calculations,

$$I_1^{\beta} = -\frac{1}{\pi^2} \left(4\frac{H_5}{\beta^4} + \frac{1}{2}\frac{H_3}{\beta^2} + \frac{H_1}{16} \right) \tag{2.27}$$

The integrals H_l, which have the same form as that of Haber and Weldon (1982), are given by

$$H_l=\frac{1}{\Gamma(l)}\int_0^\infty \frac{x^{l-1}\,dx}{(x^2+\bar{m}^2)^{1/2}} \times\left\{\frac{1}{\exp[(x^2+\bar{m}^2)^{1/2}-r\bar{m}]_{-1}}+\frac{1}{\exp[(x^2+\bar{m}^2)^{1/2}+r\bar{m}]_{-1}}\right\} \quad (2.28)$$

with $\bar{m}=a\Omega\beta$, $r=\mu/a\Omega$.

Evaluating H_l following Haber and Weldon (1982), we get

$$I_1^\beta=-\frac{\pi^2}{45\beta^4}+\frac{M^2-2\mu^2}{12\beta^2}-\frac{(M^2-\mu^2)^{3/2}}{6\pi\beta} -\frac{\mu^2}{24\pi^2}(3M^2-\mu^2)+\frac{M^4}{16\pi^2}\left(\ln\frac{4\pi}{M\beta}-\gamma\right) +O(M^6\beta^2, M^4\mu^2\beta^2)\ldots \quad (2.29)$$

It is to be noted that $|\mu|\leq a\Omega$, which is required for the same reason mentioned in Haber and Weldon (1982). Equation (2.29) contains the curvature term even in the absence of μ. Also in the limiting case our result reproduces exactly the flat-space results of Haber and Weldon (1982). Further, it can be easily verified that the integrals I_N^{FT} satisfies the relation

$$\frac{dI_N}{d\hat{\Omega}}=\hat{\Omega}(2N-1)I_{N-1} \quad (2.30)$$

with $\hat{\Omega}=a\Omega$.

For reasons discussed earlier, we renormalize the parameter following an autonomous approach (Stevenson and Tarrach, 1986) and use the following relations of Hajj and Stevenson (1988) and Stevenson and Tarrach (1986):

$$\lambda=\frac{1}{12I_{-1}(\bar{x})}, \qquad \varphi_0^2=I_{-1}(\bar{x})\Phi_0^2 \quad (2.31)$$

where $\bar{x}$ is a finite parameter bearing the dimension of mass; we normalize the bare parameters m, μ with the help of the following relations:

$$\mu^2=\frac{\mu_0^2}{I_{-1}(\bar{x})}, \qquad a^2m^2+12\lambda I_0(0)=\frac{3}{2}\frac{a^2M_0^2}{I_{-1}(\bar{x})} \quad (2.32)$$

M_0 bears the dimension of mass and μ_0 is a finite parameter and will be recognized as the chemical potential by defining it in terms of charge density ρ (Kapusta, 1981; Haber and Weldon, 1982)

$$\rho=-\frac{dV_G}{d\mu_0} \tag{2.33}$$

Minimizing V_G with respect to $\hat{\Omega}$ and rearranging, we get

$$M^2=a^2m^2+12\lambda\,(I_0+a^2\varphi_0^2)+12\lambda I_0^\beta \tag{2.34}$$

In obtaining equation (2.34) we have used the relation (2.23). Now using (2.32), we get from (2.14)

$$\frac{d\bar{V}_G}{d\varphi_0^2}=\tfrac{1}{2}a^2(M^2-8\lambda a^2\varphi_0^2+\mu^2) \tag{2.35}$$

Using (2.31) and (2.32) we can write equation (2.35) as

$$\frac{d\bar{V}_G}{d\Phi_0^2}=\frac{I_{-1}(\bar{x})}{2}\,a^2(M^2-\tfrac{2}{3}a^2\Phi_0^2)-\frac{\mu_0^2a^2}{2} \tag{2.36}$$

Again utilizing the renormalization conditions (2.31) and (2.32), we get from (2.34)

$$M^2=\tfrac{2}{3}a^2\Phi_0^2+\frac{1}{I_{-1}(\bar{x})}\,[a^2M_0^2+\Delta(\hat{\Omega},x)+\tfrac{2}{3}I_0^\beta(\hat{\Omega})] \tag{2.37}$$

In obtaining equation (2.37), we have evaluated the relations $[I_0(0)-I_0(\hat{\Omega})]$ and $[I_{-1}(\hat{\Omega})-I_{-1}(\bar{x})]$ by using the properties of the generalized ξ-function (Toms, 1980) [the method is same as that of Roy *et al.* (1989)]:

$$\xi(S,a)=\sum_{n=-\infty}^{\infty}(n^2+a^2)^{-S} \tag{2.38}$$

Using a Laurent expansion of the series appearing in (2.38) about the poles, we have

$$\xi(S,a)=D_{-1}(n,a)+D_0(n,a)+O(S+n-\tfrac{1}{2}) \tag{2.39}$$

and for D_{-1} and D_0 we have used the results of Toms (1980). Finally, the results are

$$I_0(0)-I_0(\hat{\Omega})=\frac{M^2}{2}\left[I_{-1}(\hat{\Omega})+\frac{3}{8\pi^2}\xi(3)M^2\right] \tag{2.40}$$

$$\Delta(\hat{\Omega},\bar{x})=I_{-1}(\hat{\Omega})-I_{-1}(\bar{x})$$

$$=-\frac{3\xi(3)}{4\pi^2}(a^2\Omega^2-\bar{x}^2) \qquad \text{when} \quad a\Omega\leq\bar{x} \tag{2.41}$$

$$=-\frac{1}{4\pi^2}\ln\frac{a^2\Omega^2-1}{\bar{x}^2-1} \qquad \text{when} \quad a\Omega>\bar{x} \tag{2.42}$$

With the help of equation (2.37) we get from (2.36)

$$\frac{d\bar{V}_G}{d\Phi_0^2}=\frac{a^2}{2}\left[(a^2M_0^2-\mu_0^2)+\Delta(x,y)+\tfrac{2}{3}I_0^\beta(\hat{\Omega})\right] \tag{2.43}$$

where

$$\Delta(x,y)=\frac{M^2}{4\pi^2}\xi(3)\left(\frac{M^2}{2}-\bar{x}^2\right) \qquad \text{for} \quad M\leq\bar{x}$$

$$=\frac{M^2}{12\pi^2}\left[\ln\frac{M^2}{\bar{x}^2-1}-\frac{3}{2}\xi(3)M^2\right] \qquad \text{for} \quad M>\bar{x} \tag{2.44}$$

Hence, integrating equation (2.43) with respect to Φ_0^2, we get, after subtracting the zero-point energy,

$$\bar{V}_G-D=\frac{a^2}{2}(a^2M_0^2-\mu_0^2)\Phi_0^2+\Gamma(\hat{\Omega},x)+I_1^\beta \tag{2.45}$$

with

$$\Gamma(\hat{\Omega},x)=\frac{3}{32\pi^2}\xi(3)M^4\left(\frac{M^2}{3}-x^2-1\right) \qquad \text{for} \quad M\leq x$$

$$=\frac{M^4}{32\pi^2}\left[\ln\frac{M^2}{x^2}-\frac{1}{2}-\xi(3)M^2\right] \qquad \text{for} \quad M>x \tag{2.46}$$

where $x^2=\bar{x}^2-1$ and D is the constant of integration resulting from the solution of (2.43). While performing the integration, we have used the fact that $M^2\simeq\frac{2}{3}a^2\Phi_0^2$ [cf. (2.37)]. Following the arguments and steps of Hajj and Stevenson (1988), it can be easily verified that the constant D appearing in equation (2.45) is nothing but the zero-point energy.

3. SPONTANEOUS SYMMETRY BREAKING WITH FIXED CHARGE DENSITY

The system under consideration has a background charge introduced by ρ, which in turn is related to μ_0 via equation (2.33). Using (2.45), we have from equation (2.33)

$$\rho = -\left(\frac{dI_1^\beta}{d\mu_0}\right)_{(T,M)\text{constant}} + \mu_0\Phi_0^2 \tag{3.1}$$

From equation (3.1) it is obvious that the value of μ_0 for a given Φ_0 can be obtained for a fixed charge density ρ, since I_1^β can be expressed in terms of Φ_0 ($M^2 \simeq \frac{2}{3}a^2\Phi_0^2$). Thus, $\bar{V}_G$ can be computed [cf. (2.45), (2.46)] for different Φ_0 after obtaining μ_0 for a given charge density from (3.1).

The computation results show that the nature of the $\bar{V}_G$–Φ_0 curve does not depend on a^2 qualitatively. In this regard our conclusion is the same as that of others (Roy *et al.*, 1989; Tarrach, 1986) (since a^2 may be absorbed in bare parameters). Figure 1 shows the variation of $\bar{V}_G$ with Φ_0 at different

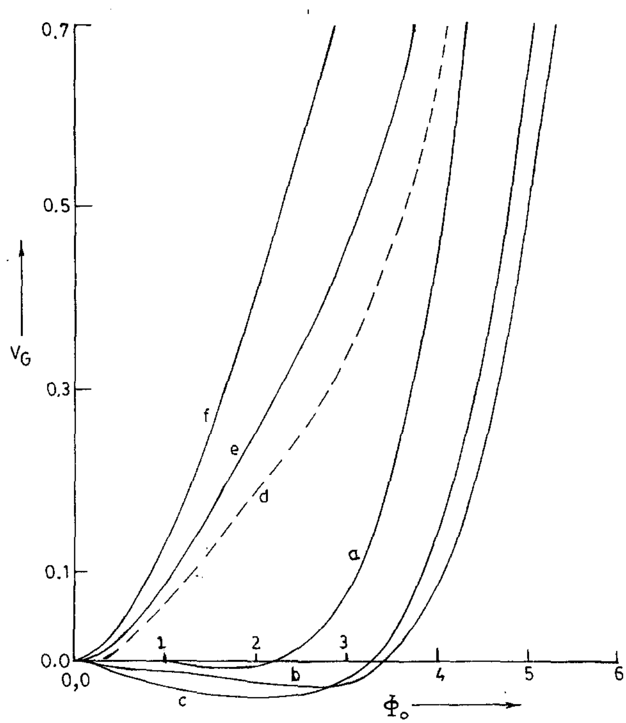

Fig. 1. $\bar{V}_G$–Φ curve for $a^2 = 1$, $M_0^2 = 0$, $\rho = 0.001$, and (a) $T = 0$, (b) $T = 0.5$, (c) $T = 0.7$, (d) $T = T_c = 1.35$, (e) $T = 1.5$, (f) $T = 1.8$.

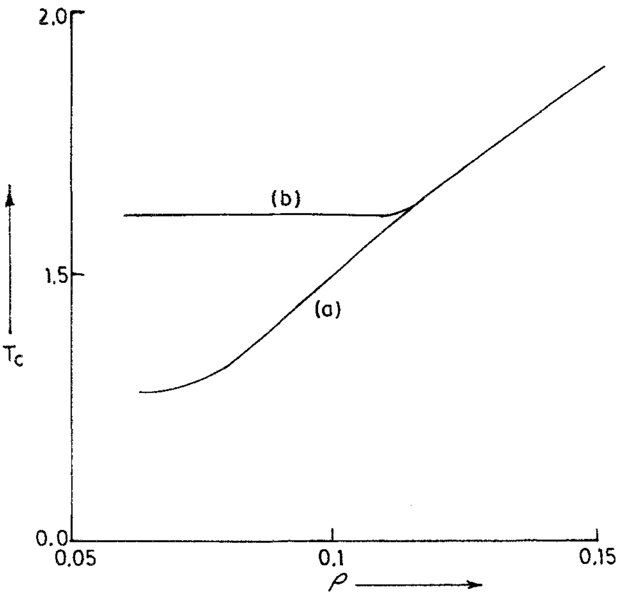

Fig. 2. ρ–T curve for $a^2 = 1.0$ and (a) $M_0^2 = -0.05$, (b) $M_0^2 = -0.1$.

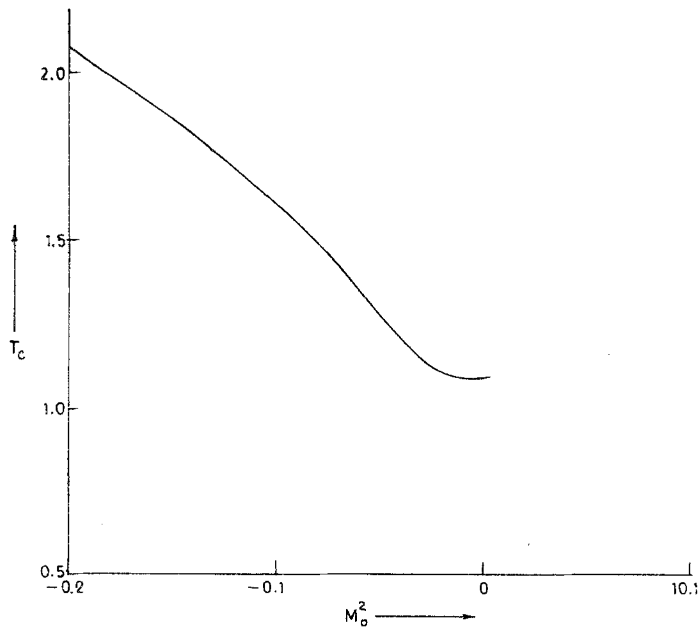

Fig. 3. M_0^2–T_c curve for $\rho = 0.001$, $a^2 = 1.0$.

temperatures. A temperature-dependent constant term is added to each curve so that all curves coincide at the origin. For the computation of Figure 1 we have taken $x=1$, $M_0^2=0$, $\rho=0.001$, and $a^2=1$. The critical temperature T_c for Figure 1 is 1.35. We have not presented $\bar{V}_G$–Φ_0 curves for different a^2 for reasons already mentioned. Again it is observed (Figure 2) that for M_0^2, a^2 remaining fixed, T_c increases with increasing value of ρ. Also it is noted that for different masses ($M_0^2=-0.05$, -0.1), the critical temperatures do not vary after a certain value of ρ ($=0.12$).

Finally, Figure 3 shows the variation of T_c with M_0^2. It is interesting to note that for $M_0^2>0$ the symmetry breaking does not occur and T_c increases with M_0^2, acquiring a more and more negative value. Moreover, our result reproduces the flat-space results of Hajj and Stevenson (1988) in the limiting case with $\mu=0$.

4. DISCUSSION AND REMARKS

We have chosen an autonomous GEP approach instead of a loop expansion method since the former approach has several advantages. Also it has been observed (Hajj and Stevenson, 1988) that loop expansion does not give the correct high-temperature behavior. Moreover, we see that even in curved space with a background charge, a finite-temperature $\bar{V}_G$ appears in a simple form and the symmetry restoration occurs at high temperature. Finally, we add that we have considered a single μ. However, the extension of the present case to more than one μ is straightforward.

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